The Blume–Emery–Griffiths model at an infinitely many ground states interface and exponential decay of correlations at all nonzero temperatures

2004 J. Phys. A: Math. Gen. 37 1459

(http://iopscience.iop.org/0305-4470/37/4/027)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **37** (2004) 1459–1460 PII: S0305-4470(04)70387-X

ADDENDUM

The Blume–Emery–Griffiths model at an infinitely many ground states interface and exponential decay of correlations at all nonzero temperatures

Gastão A Braga and Paulo C Lima

Departamento de Matemática, UFMG, Caixa Postal 1621, 30161-970 Belo Horizonte, MG, Brazil

Received 9 October 2003
Published 9 Janauary 2004
Online at stacks.iop.org/JPhysA/37/1459 (DOI: 10.1088/0305-4470/37/4/027)

Abstract
We present a corrected proof of theorem 1.1 of our previous paper (Braga G A and Lima P C 2003 *J. Phys. A: Math. Gen.* **36** 9609) and extend it to a subset of parameter values in the disordered region.

PACS numbers: 05.70.F, 64.60, 64.70, 07.20.M

Due to the wrong definition of b in equation (9) of [1], the proof of theorem 1.1 presented in section 3 of this reference is incorrect. The definition of b has no parenthesis on it, i.e., $b \equiv \mathrm{e}^{\beta y}\cosh\beta - 1$, and therefore b is not positive if $y < -1$, a statement that the authors have used in the course of the proof. The aim of this addendum is to present the correct proof of theorem 1.1. Its statement remains the same but the convergence condition (2) is replaced by another one which will be determined below. At the end of this addendum we extend the proof to a subset of the parameter values x and y in the disordered region $\{(x, y) : x < 0, 1 + 2x + y < 0\}$.

Proof of theorem 1.1. As in [1], we consider the interface $x = 0$ and $y < -1$. We replace (15) of [1] by

$$\langle\sigma_i\sigma_j\rangle_\Lambda^0 = \sum_{\Gamma_1\supset\{i,j\}} \mathrm{e}^{\beta y|\Gamma_1^*|}\left(\sum_{\{\sigma\}\in\{-1,1\}^{|\Gamma_1|}} \sigma_i\sigma_j \prod_{(k,l)\in\Gamma_1^*} \mathrm{e}^{\beta\sigma_k\sigma_l}\right)\frac{Z^0_{\Lambda-(\Gamma_1\cup\partial\Gamma_1)}}{Z^0_\Lambda} \tag{1}$$

where Γ_1 is a one-connected subset of Λ containing $\{i, j\}$ and the superscript 0 means 0-boundary condition (which is equivalent to the free-boundary condition in the considered interface, see [1]). Identity (1) holds true because of the spin flip symmetry of the model, namely, for any configuration giving a nonzero contribution to the numerator of the expected value, the spin variables σ_i and σ_j must be connected to each other by a path where the spin values are +1 or −1.

We first analyse (1) at high temperature, uniformly in y. Equation (1) can be written as

$$\langle\sigma_i\sigma_j\rangle_\Lambda^0 = \sum_{\Gamma_1\supset\{i,j\}} \langle\sigma_i\sigma_j\rangle^I_{\Gamma_1}\mathrm{e}^{\beta y|\Gamma_1^*|}Z^I_{\Gamma_1}\frac{Z^0_{\Lambda-(\Gamma_1\cup\partial\Gamma_1)}}{Z^0_\Lambda} \tag{2}$$

where the index I stands for the d-dimensional Ising model with free-boundary condition. Since $\Gamma_1 \subset \Lambda$, the Griffiths inequalities (see section 4.1 of [2]) imply that $\langle\sigma_i\sigma_j\rangle^I_{\Gamma_1} \leqslant \langle\sigma_i\sigma_j\rangle^I_\Lambda$ and from (2) we get that

$$\langle\sigma_i\sigma_j\rangle^0_\Lambda \leqslant \langle\sigma_i\sigma_j\rangle^I_\Lambda \frac{\sum_{\Gamma_1\supset\{i,j\}} \mathrm{e}^{\beta y|\Gamma_1^*|} Z^I_{\Gamma_1} Z^0_{\Lambda-(\Gamma_1\cup\partial\Gamma_1)}}{Z^0_\Lambda}$$

$$\equiv \langle\sigma_i\sigma_j\rangle^I_\Lambda \frac{\tilde{Z}_\Lambda}{Z^0_\Lambda} \leqslant \langle\sigma_i\sigma_j\rangle^I_\Lambda \qquad (3)$$

where we have used that $\tilde{Z}_\Lambda Z_\Lambda^{-1} \leqslant 1$ because $\tilde{Z}_\Lambda$ is obtained from Z^0_Λ by restricting it to those configurations σ, for which there is a one-connected subset $\Gamma_1 \subset \Lambda$ containing i and j and $\sigma_{\Gamma_1} \in \{-1, 1\}^{|\Gamma_1|}$. The Ising spin–spin correlation function decays exponentially if $\beta < \beta^I_d$, where β^I_d is given by the relation $4d^2\left(\exp\beta^I_d - 1\right) = 1$ (this condition is obtained using, for instance, the argument given in section 3 of [1]), implying that $\langle\sigma_i\sigma_j\rangle^0_\Lambda$ decays exponentially if $\beta < \beta^I_d$ and $y \in \mathbb{R}$.

Now we analyse (1) in the regime $\beta \geqslant \beta^I_d$. Using that the product $\left(Z^0_\Lambda\right)^{-1} Z^0_{\Lambda-(\Gamma_1\cup\partial\Gamma_1)}$ is bounded above by 1, that $|\Gamma_1| \leqslant 2|\Gamma_1^*|$ and taking $y < -1$, we get from (1)

$$\langle\sigma_i\sigma_j\rangle^0_\Lambda \leqslant \sum_{\Gamma_1\supset\{i,j\}} 2^{|\Gamma_1|}\mathrm{e}^{\beta(y+1)|\Gamma_1^*|} \leqslant \sum_{\Gamma_1\supset\{i,j\}} 2^{2|\Gamma_1^*|}\,\mathrm{e}^{(y+1)\beta|\Gamma_1^*|}. \qquad (4)$$

As at the end of section 3 of [1], the last sum in (4) is bounded above by

$$\sum_{n\geqslant\|i-j\|} \left(16d^2\,\mathrm{e}^{(y+1)\beta^I_d}\right)^n \qquad (5)$$

which is convergent (and decays exponentially with the distance $\|i - j\|$) as long as $y < y_d$, where y_d is defined by the relation $16d^2 \exp\left[(y_d + 1)\beta^I_d\right] = 1$, completing the proof. □

Using now the Hamiltonian $H(\sigma) = -\sum_{(i,j)} \left[\sigma_i\sigma_j + y\sigma_i^2\sigma_j^2 + x\left(\sigma_i^2 + \sigma_j^2\right)\right]$, the above analysis can be extended to a subset of the disordered region $\{(x, y) : x < 0, 1 + 2x + y < 0\}$. We observe that (1), (2), (3) and the first inequality in (4) still hold if the exponential factor $\exp(\beta y|\Gamma_1^*|)$ is replaced by $\exp[\beta(2x + y)|\Gamma_1^*| + \beta x|\partial\Gamma_1|]$. The last inequality in (4) and inequality (5) hold if $x \leqslant 0$ and if we replace y by $2x + y$. Therefore, we have extended theorem 1.1 of [1]:

Theorem 1. *The spin–spin correlation function of the d-dimensional BEG model with Hamiltonian* $H(\sigma) = -\sum_{(i,j)} \left[\sigma_i\sigma_j + y\sigma_i^2\sigma_j^2 + x\left(\sigma_i^2 + \sigma_j^2\right)\right]$ *decays exponentially if* (i) $x, y \in \mathbb{R}$ *and* $\beta < \beta^I_d$ *or* (ii) $x \leqslant 0$, $\beta \geqslant \beta^I_d$ *and* $2x + y < y_d$, *where* β^I_d *and* y_d *are defined as above.*

References

[1] Braga G A and Lima P C 2003 *J. Phys. A: Math. Gen.* **36** 9609

[2] Glimm J and Jaffe A 1987 *Quantum Physics, A Functional Integral Point of View* 2nd edn (New York: Springer)